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# **Zero Rating and Capacity Investment**

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## Abstract

This paper analysis zero-rating, a practice where Mobile Network Operators (MNOs) exclude some traffic from being charged when consumers use mobile broadband data. Focusing on the downstream effects and considering two competing firms, two contents and network effects (externality) on one of the contents, it is shown that zero-rating can be used to increase, and thereafter, extract more surplus from consumers, when network effects are strong enough. Moreover, it is shown that the interaction between contents has an impact on investment decisions, as substitutes contents lead to lower investment when the network effect increases while complementary contents result in higher investment for stronger network effects. Finally, zero-rating can lead to higher network capacity than under joint billing, when investment costs are high enough.

**Keywords:** Zero-rating, network effects, capacity constraints.

# I Introduction

Recent years have brought many changes to mobile internet usage, both in terms of contents and services available and amount of data transferred. Nonetheless, this has caused mobile network operators (MNOs) to face an ever-increasing demand for data that they have been trying to match with capacity investment. In addition, online services such as WhatsApp or Skype are reducing demand for traditional mobile services (i.e. SMSs and calls), leading to a decrease in MNOs' revenue through two sources: the usage of such services and termination rates. Therefore, there is a decoupling between the revenue streams from provision of content and ownership of the network, as pointed out by Baldry et al. (2014). This decoupling results in uncertainty over revenue generation for MNOs, driving lower incentives to invest in capacity.

MNOs have been implementing network management practices in order to overcome the increased uncertainty over revenue generation. One of the most recent is zero rating, in which traffic generated by specific apps or websites is exempt from any usage charges. With such schemes, MNOs expect to increase consumer surplus from subsidizing certain applications and/or online services and charge them accordingly, increasing revenues and thus, getting higher returns on their capacity investments. In fact, this is the main argument used by MNOs to justify zero-rating practices.

Much of the discussion on this topic deals with the content discrimination provoked by zero rating as a violation of net neutrality rules, and its impact on the content market (upstream market), namely, on the harmful effects that it has on competition among content providers. However, it is also important to assess the impact of zero rating on the downstream market. While there have been some studies on this,<sup>1</sup> they focus on a monopolist MNO in different settings. As such, the focus of this paper will be in a competitive setting, with homogeneous consumers and network effects on one of the contents, and the main goal is to analyse how capacity investment is affected by zero

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<sup>1</sup>As discussed in section II.

rating under competition.

The paper is organized as follows: section II will discuss related literature; section III will describe the framework to be used in the analysis; section IV will analyse the impacts on investment incentives; section V present the case of a monopolist; and section VI concludes.

## **II Literature Review**

This paper relates most to those analysing zero rating practices. As mentioned before, most of the literature focuses on upstream effects of zero rating. As an example, Jullien et al. (2016) analyse how MNOs can use data sponsorship plans to increase revenues. Nonetheless, the European Commission (2014) reports that there is little evidence of such deals being struck as well as the limited number of related complaints. In addition, van Schewick (2015) argues that zero-rating is potentially a violation of net neutrality.

Some papers take a closer look at the downstream effects, and more specifically, at investment incentives. For instance, Somogyi et al. (2017) uses a monopolist MNO facing a two-sided market with homogeneous consumers and two competing content providers to show that zero rating can be optimal if contents are very unattractive or very attractive, but not in an intermediate region. The subsequent impact on welfare is either negative for unattractive content or positive for attractive content. Moreover, Preta and Peng (2015) find that, while zero-rating could have anti-competitive consequences, they reckon that it is potentially welfare enhancing based on network effects, price discrimination, two-sided market and behavioural economics explanations. Along with this, Eisenach (2015) also describes the economic foundations of zero-rating and its concerns, and concludes that zero-rating is a mechanism to capture economic efficiencies, while rejecting it as an anti-competitive strategy.

Inceoglu and Liu (2017) look at a monopolist MNO facing heterogeneous consumers, and show that zero rating can be used as a price discrimination tool to better differentiate between consumer types. They found that generally, zero rating can increase welfare if contents are not close substitutes and preferences are sufficiently different. More interestingly, they found that zero rate increases capacity investment, if costs are not too high. This paper also contributes to the discussion on net neutrality, for which Greenstein et al. (2016) summarizes the main findings and arguments used in the debate.

More generally, I consider Hotelling competition with capacity constraints, which was previously looked at by Wauthy et al. (1996), where he showed that capacity constraints may restore the existence of equilibrium with pure strategies when products are too similar. The intuition of this paper is that when products are similar, firms have incentives to undercut each others prices to grab the whole market (similar to what happens in Bertrand competition), however the existence of capacity constraints means that firms cannot serve the whole market and therefore, incentives to undercut prices disappear once capacity is reached.

### III Framework

To analyse investment decisions under zero rating, the following setting is proposed: there are two contents  $x$  and  $y$ , accessible through an internet service provided by one of two competing operators. Operators charge a fixed fee  $F$  and provide consumers with a data cap (allowance of data)  $q$ , which cannot exceed the per consumer capacity allocation  $k$ . The latter is a function of capacity  $Q$  and the number of consumers  $N$  the MNO wants to serve, such that  $k = \frac{Q}{N}$ . Firms have constant marginal cost  $c$  of investing in capacity, but using capacity is costless. Moreover, I assume that there is a positive network effect on content  $x$ , that is, the more  $x$  is consumed, the better off consumers are. This means that if there is an incentive to zero rate, it must be on content  $x$  and both

firms will want to do it.

### III.A Model Setup

In this setting, I have a mass 1 of consumers with homogeneous preferences over contents obtained via a mobile broadband connection. Furthermore, it is assumed that there is satiation in consumption of both contents, i.e. marginal utility of consumption goes to zero. This is specially important under zero rating where one of the contents will be unconstrained. Without this assumption, optimal consumption of  $x$  would be infinite without an additional constraint (i.e. time constraint), which would add unnecessary complications to the model and subsequent analysis. Consumers choice of network is modelled below. For a given choice of network, the utility function, for a consumer  $n$ , used for the analysis is:

$$S(x_n, y_n, X) = x_n + y_n - \frac{1}{2}x_n^2 - \frac{1}{2}y_n^2 - \gamma x_n y_n + \beta X,$$

where  $\gamma \in ]-1, 1[$  is a parameter for the interaction between contents

$\beta > 0$  is a parameter for the strength of the network effect

$X$  represents total use of content  $x$  on the network that consumer  $n$  connected to.

For values of  $\gamma > 0$  contents are substitutes, whereas for values of  $\gamma < 0$  contents are complements. Moreover, consumers, when maximizing utility over contents (not network choice), do not take into account the externality, which creates inefficiency in consumption.

Consumers face a usage constraint  $y + \lambda r x \leq q$  imposed by the MNO, where  $r$  is an exogenous parameter that measures how much bandwidth is used per unit of  $x$  relative to  $y$  (which has bandwidth usage normalized to 1). In this model, bandwidth reflects the implicit price of consuming content, i.e. how much data is displaced by consuming either  $x$  or  $y$ .  $\lambda \in [0, 1]$  is a parameter for how MNOs "price"<sup>2</sup> content  $x$ . For  $\lambda = 1$  consumption of  $x$  counts towards the cap (henceforth,

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<sup>2</sup>Note that content  $y$  will always have  $\lambda_y = 1$ .

referred to as joint billing) and for  $\lambda = 0$  the content is said to be zero rated (consumption of  $x$  does not count towards the data cap). For any other value, there is partial billing; however I will only focus on joint billing vs. zero rating.

The consumer maximization problem is then:

$$\max_{x,y} U(x,y) = x + y - \frac{1}{2}x^2 - \frac{1}{2}y^2 - \gamma xy \quad (1)$$

$$s.t. y + \lambda r x \leq q. \quad (2)$$

As mentioned before, this model deals with two firms (MNOs). Given their market share  $\alpha$ , profit functions are  $\pi_i = \alpha F_i - cQ_i$  subject to  $Q_i = \alpha k_i$ . Firms set capacity  $Q_i$ , fixed fee  $F_i$  and do not charge for usage, as marginal costs of using capacity is zero and the data cap ensures that consumption does not exceed capacity. The per consumer capacity allocation follows implicitly from the fixed fee and the resulting market share.

Similar to what is done in Laffont et al. (1998), competition occurs on a Hotelling line [0,1], where each firm is located at one end of the line and consumers are distributed through the line. Their position  $p$  on the line reflects their preferential network characteristics and  $p_i$  represents firms' locations. Consumers get surplus  $S_i(k_i)$  from connecting to network  $i$  and have a cost  $t$  from not connecting to their preferential network. Given their position  $p$ , capacity allocated to them  $k_i$  and fixed fee  $F_i$ , consumers get utility from connecting to network  $i$ :

$$S_i(k_i) - F_i - t|p - p_i|.$$

Let  $w_i = S_i(k_i) - F_i$  be the consumers' net utility from connecting to network  $i$ . A consumer located at distance  $\alpha$  from firm  $i$  is indifferent between networks if:

$$w_i - t\alpha = w_j - t(1 - \alpha).$$

Solving for  $\alpha$ , the market share of firm  $i$  is obtained:

$$\alpha = \frac{1}{2} + \frac{1}{2t}(w_i - w_j). \quad (3)$$

Decisions are taken in the following order: firstly, firms choose capacity; then, firms decide fixed fees and whether to zero rate consumption of content  $x$  or not; consumers then choose which network they are connecting to; and finally, consumers choose how much of contents  $x$  and  $y$  they will consume. I find the subgame-perfect Nash equilibrium.

### III.B Consumer Choice and Market Equilibrium

#### III.B.1 Consumers' Decision

First I look at the optimal consumption levels of  $x$  and  $y$  for a given choice of network. However, as mentioned before, the utility function considered has a satiation point. As such, consumer choice has to be analysed under a binding and non-binding cap.

**Non-binding cap** For large enough data caps, consumption becomes unconstrained (equivalent to having no data cap). Maximizing equation (1) without any constraints results in optimal consumption levels:

$$x^* = \frac{1}{1+\gamma}, \quad y^* = \frac{1}{1+\gamma},$$

$$U(x^*, y^*) = \frac{1}{1+\gamma}.$$

Total consumption is  $rx + y = \frac{1+r}{1+\gamma} \equiv K$  and consumption that counts towards the cap is  $\lambda rx + y = \frac{1+r\lambda}{1+r}$ . Thus, for any  $q \geq \frac{1+r\lambda}{1+r}$ , the cap is non-binding.

**Binding cap** Let  $y + \lambda rx = q$ , thus  $y = q - \lambda rx$ . Consumers maximize surplus<sup>3</sup>  $U(x, q - \lambda rx)$  resulting in  $x = \frac{1-q\gamma-r\lambda+qr\lambda}{1+r^2\lambda^2-2r\lambda\gamma}$ .

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<sup>3</sup>  $\frac{\partial^2 U}{\partial x^2} = -1 - r^2\lambda^2 + 2r\lambda\gamma < 0$  is true since  $\gamma < 1$ .



With this, it is possible to write  $q$  as a function of per customer capacity usage  $k$ , by solving  $rx + q - \lambda rx = k$  for  $q$ :

$$q = \frac{1 + r^2\lambda^2 - 2r\lambda\gamma}{1 - r\lambda\gamma - r\gamma + r^2\lambda}k + \frac{-r^2\lambda^2 + r\lambda + r^2\lambda - r}{1 - r\lambda\gamma - r\gamma + r^2\lambda}. \quad (4)$$

One would expect that  $q$  be increasing in  $k$ . A sufficient condition for this is  $1 - r\gamma > 0 \Leftrightarrow r\gamma < 1$ , which is assumed henceforth.<sup>4</sup>

Moreover, in the remainder of the paper it is assumed that the cap is binding. Without this assumption, zero rating would not have any impact on consumption decisions.

### III.B.2 Firms' Decision

Since firms charge a two part tariff, where the usage price is zero due to zero marginal costs, and network effects are limited to each network, the choice of zero-rating can be decoupled from the choice of fixed fees. Thus, the choice of zero-rating is analysed first, and fixed fees and equilibrium market shares are determined subsequently.

**Conditions for Zero Rating** Firms are aware of the network effects, thus their profit maximization problem takes into account the changes in consumer surplus that are due to this. By increasing the surplus, firms expect to be able to capture it with higher fixed fees and/or end up with higher market share. As such, in a first step, firm  $i$  maximizes surplus from consumption only considering  $\lambda$  as a decision variable. Since consumers have homogeneous preferences over contents, then  $x_n = x$  and  $y_n = y$ . With  $x$  and  $y$  from the previous section and  $X = Nx$ , surplus from consumption becomes:

$$\max_{\lambda} S(x, y) = x + y - \frac{1}{2}x^2 - \frac{1}{2}y^2 - \gamma xy + \beta Nx. \quad (5)$$

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<sup>4</sup> $r\gamma < 1 \Rightarrow 1 - r\lambda\gamma - r\gamma + r^2\lambda, \forall \lambda \in [0, 1]$ .

The surplus maximizing  $\lambda$  can be found as:

$$\lambda = \frac{r - N\beta - r\gamma - r^2\gamma - kr + r^2 + kr\gamma^2 + Nr\beta\gamma}{r - r\gamma - r^2\gamma - kr + r^2 + Nr^2\beta + kr\gamma^2 - Nr\beta\gamma}. \quad (6)$$

We have a maximum<sup>5</sup> at  $\lambda = 0$  when  $\beta = r(1 - \gamma^2) \frac{K-k}{N(1-r\gamma)} \equiv \beta^*$ . For  $\beta > \beta^*$ ,  $\lambda = 0$  is a boundary maximum.

**Proposition 1** *Zero-rating is optimal if network effects are strong enough, i.e.  $\beta N \geq r \frac{1-\gamma^2}{1-r\gamma} (K - k)$ .*

At  $\lambda = 0$  (zero rating), the cap becomes  $q = \frac{k-r}{1-r\gamma}$  and the consumer maximization yields:

$$y_{zr}^* = \frac{k-r}{1-r\gamma}, \quad x_{zr}^* = \frac{1-\gamma k}{1-r\gamma},$$

$$U(x^*, y^*) = \frac{1}{1+\gamma} - \frac{1-\gamma^2}{2(1-r\gamma)} (K-k)^2.$$

To have positive consumption<sup>6</sup> it must be that  $k - r \geq 0 \Rightarrow k \geq r$  and  $1 - \gamma k \geq 0 \Rightarrow k \leq \frac{1}{\gamma}$ . Notice however that this last condition is trivial since  $k \leq K$  is needed and that  $K \leq \frac{1}{\gamma}$  is always true when  $r\gamma < 1$ . Thus the interval for eligible values of  $k$  is:

$$r \leq k \leq K,$$

which is non-empty since  $r < K$  when  $r\gamma < 1$ .

For simplicity I will use  $B = \frac{1}{1+\gamma}$  and  $A = \frac{1-\gamma^2}{2(1-r\gamma)}$  such that surplus becomes:

$$S_{zr}(k) = B - A(K-k)^2 + \beta N x_{zr}^*. \quad (7)$$

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<sup>5</sup>SOC:  $\frac{\partial^2 S(x, y, X)}{\partial \lambda^2} < 0$  when  $1 - 2r\gamma + r^2 > 0$ .

<sup>6</sup>Recall that  $r\gamma < 1$  is needed to sustain zero rating.

Analogously, at  $\lambda = 1$  (joint billing), the cap is  $q = k$  and maximization of surplus yields:

$$y_{jb}^* = \frac{k - r + r^2 - k\gamma r}{r^2 - 2r\gamma + 1}, \quad x_{jb}^* = \frac{1 - r - k\gamma + kr}{r^2 - 2r\gamma + 1},$$

$$U(x_{jb}^*, y_{jb}^*) = \frac{1}{1 + \gamma} - \frac{1 - \gamma^2}{2(1 - 2r\gamma + r^2)} (K - k)^2.$$

In this case, it is necessary that  $1 - r - k\gamma + kr \geq 0 \Rightarrow k \leq \frac{1-r}{\gamma-r}$  and  $k - r - r^2 - k\gamma r \geq 0 \Rightarrow k \geq r \frac{1-r}{1-\gamma r}$ . However,  $k \leq \frac{1-r}{\gamma-r}$  is trivial since  $\frac{1-r}{\gamma-r} > \frac{1+r}{1+\gamma} = K$ . As such, the eligible region of analysis is:

$$r \frac{1-r}{1-\gamma r} \leq k \leq K.$$

Furthermore, for  $\gamma < 1 \Rightarrow r \frac{1-r}{1-\gamma r} < r$ , hence the analysis can be restricted to  $k \geq r$  without further issues.

Letting  $G = \frac{1-\gamma^2}{2(1-2r\gamma+r^2)}$ , surplus is:

$$S_{jb}(k) = B - G(K - k)^2 + \beta x_{jb}^*. \quad (8)$$

**Equilibrium Fixed Fees and Market Shares** For a given level of capacities  $(Q_i, Q_j)$ , firm  $i$ 's maximization problem is:

$$\begin{aligned} \max_{k_i, w_i} \pi_i &= \alpha (S_i(k_i) - w_i) - cQ_i \\ s.t. \quad Q_i &= \alpha k_i \end{aligned} \quad (9)$$

(3).

From first order conditions<sup>7</sup> it follows the reaction function of firm  $i$ 's net utility:

$$w_i = -\frac{t}{2} + \frac{1}{2}S_i(k_i) + \frac{1}{2}w_j - \frac{1}{2}\frac{\partial S_i}{\partial k_i}k_i. \quad (10)$$

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<sup>7</sup>See appendix A for computations.

Substituting back the analogous firm  $j$ 's reaction function and replacing  $w_i = S_i(k_i) - F_i$ , the resulting Nash equilibrium fixed fees are:

$$F_i = t + \frac{1}{3}S_i(k_i) - \frac{1}{3}S_j(k_j) + \frac{2}{3}\frac{\partial S_i}{\partial k_i}k_i + \frac{1}{3}\frac{\partial S_j}{\partial k_j}k_j. \quad (11)$$

The differences from the standard Hotelling model are the  $\frac{2}{3}\frac{\partial S_i}{\partial k_i}k_i$  and  $\frac{1}{3}\frac{\partial S_j}{\partial k_j}k_j$ , both affecting positively the fixed fee. It can be argued that beyond the direct effect that  $k_i$  and  $k_j$  have on consumers' utility (which are the same as in the standard Hotelling model), increasing the capacity per customer restricts the amount of consumers the firm can supply, therefore the new "last" consumer is closer to the firm (in the Hotelling line) and thus willing to pay more than the previous "last" consumer, resulting in higher fixed fee. Conversely, if the other firm increases capacity per customer, then it restricts the amount of consumers it can serve, increasing the market power of the original firm over a bigger segment of the Hotelling line, also resulting in higher fixed fees.

The equilibrium market share of firm  $i$  then becomes:

$$\alpha = \frac{1}{2} + \frac{1}{6t} \left( S_i(k_i) - S_j(k_j) - \frac{\partial S_i}{\partial k_i}k_i + \frac{\partial S_j}{\partial k_j}k_j \right). \quad (12)$$

Increases in  $k_i$  should have a positive impact on firm  $i$ 's market share. This can be shown by

$$\frac{\partial \alpha}{\partial k_i} = -\frac{1}{6t} \frac{\partial^2 S_i}{\partial k_i^2} k_i > 0.$$

## IV Investment Decisions

Replacing  $k_i = \frac{Q_i}{\alpha}$  and  $k_j = \frac{Q_j}{1-\alpha}$ , given by the original constraints, in the firm's maximization problem results in a 5th order equation in  $\alpha$ , which cannot be solved analytically, as demonstrated by Abel (1824). Hence, in order to solve the model it is necessary to look for a symmetric equilibrium in capacities  $Q_i$ . Namely, if it is considered that investment costs are the same for both firms, then in equilibrium they will invest the same amount as well as provide the same data cap

and charge the same fixed fee, resulting in  $\alpha = \frac{1}{2}$ . Moreover, it is important to keep in mind that  $k$  is bounded, therefore, there will also be bounds on  $c$  that can be considered for the analysis. Hence, the analysis will be done for values of  $r < k < K$  or equivalently  $\frac{r}{2} < Q < \frac{K}{2}$ .

Letting  $b = \frac{\beta}{1-r\gamma}$  and  $v = \frac{\beta}{r^2-2r\gamma+1}$ , firm  $i$ 's profit functions become:

$$\pi_i^{zr} = \alpha (B - A(K - k_i)^2 + \alpha b(1 - \gamma k_i) - w_i) - cQ_i, \quad (13)$$

$$\pi_i^{jb} = \alpha (B - G(K - k_i)^2 + \alpha v(1 - \gamma k_i - r + rk_i) - w_i) - cQ_i, \quad (14)$$

for zero-rating and joint billing, respectively. Still holding capacities  $Q_i$  and  $Q_j$  constant, solving the model using the approach described in section III.B.2 yields equilibrium fixed fees:

$$F_i^{zr} = -2Ak_1^2 + 2AKk_1 + \alpha(2t - b), \quad (15)$$

$$F_i^{jb} = -2Gk_1^2 + 2GKk_1 + \alpha((r - 1)v + 2t). \quad (16)$$

Replacing equations (15), (16) and  $\alpha k_i = Q_i$  in the firms' profit function  $\alpha F_i - cQ_i$ , it is possible to take first order conditions<sup>8</sup> over  $Q_i$ . Imposing a symmetric equilibrium results in investment curves:

$$c = \frac{-8A^2k^3 + (8A^2K - 6\gamma bA)k^2 + (2\gamma bAK + 12bA - 16At)k + (12AKt - 8bAK + \gamma b^2 - 2\gamma bt)}{6t + 4k^2A + b\gamma k - 4b}, \quad (17)$$

$$c = \frac{(\gamma - r + r^2 - r\gamma)v^2 - 8G^2k^3 + (12Gk + 2rt + 6Gk^2r + 8GKr + 2GKk\gamma - 12Gkr - 2t\gamma - 8GK - 6Gk^2\gamma - 2GKkr)v + 8G^2Kk^2 + (12GK - 16Gk)t}{6t - 4v + 4Gk^2 + 4rv + kv\gamma - krv}, \quad (18)$$

for zero-rating and joint billing, respectively. The curves cannot be compared analytically, therefore, I will set parameter values that allow for such comparison. Using parameter values  $\gamma = 0.5$ ,  $r = 0.5$ ,  $\beta = 0.18$  and  $t = 2$  investment curves are:

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<sup>8</sup>See appendix A for derivation.

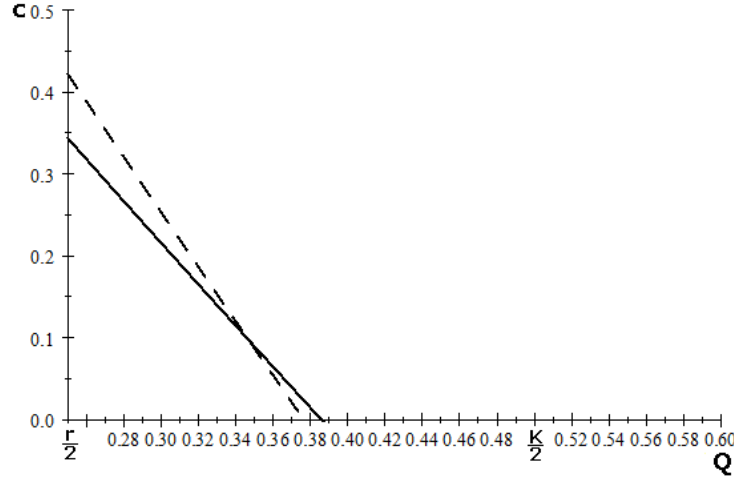


Figure 1: Investment curves with parameters:  $\gamma = 0.5$ ,  $r = 0.5$ ,  $\beta = 0.18$  and  $t = 2$ ; Dashed line: Zero-rating; Solid line: Joint billing

with solid and dashed lines representing investment decision under joint billing and zero rating, respectively.

For the proposed parameter values, one quickly concludes that there is a region where investment in capacity will be lower under zero rating, namely when investment costs are low enough. This means that the MNOs' argument that zero rating increases capacity investment is not clear and unconditional.

One important result is the impact that the interaction between contents has on the investment decisions. While the sign of  $\gamma$  has no impact on the result stated above, it does influence how MNOs react to changes in the strength of network effects. In the case of zero-rating, for  $\gamma > 0$  (substitute contents), increasing the strength of the network effect  $\beta$  reduces the incentives to invest in capacity.<sup>9</sup> In fact, by analysing  $\frac{\partial^2 \pi_i^{zr}}{\partial k_i \partial \beta} = \alpha^2 \frac{-\gamma}{1-r\gamma}$  it is evident that for substitute contents, increased strength of network effect reduces the marginal benefit of increasing  $k_i$  and, implicitly, increasing  $Q_i$ . On the other hand, if contents are instead complements (i.e.  $\gamma < 0$ ) the opposite is true.<sup>10</sup> The

<sup>9</sup>See Figure 3 in appendix B for investment decisions with stronger network effects.

<sup>10</sup>See figures 4 and 5 in appendix B for an example of this.

same is true under joint billing, namely when  $r < \gamma$ , as  $\frac{\partial^2 \pi_i}{\partial k_i \partial \beta} = \alpha^2 \frac{r-\gamma}{r^2-2r\gamma+1}$ .

**Proposition 2** *Increasing the strength of the network effect: (i) increases the incentives to invest in capacity if contents are complements; (ii) decreases incentives to invest if contents are substitutes.*

## V Monopoly case

To see the effects of competition on investment decisions, it is important to have the monopolist case as a benchmark. As such, the same model is derived without firm  $j$ . The consumer with location  $p = \alpha$  is now indifferent between opting to join the network or not when  $S(k_i) - F_i - t\alpha = 0$ , where 0 is the reservation utility (outside option). For simplicity, I look at the case where profits are maximized at full coverage, i.e.  $\alpha = 1$ , resulting in  $F_i = S(k_i) - t$  and  $k_i = Q_i$ . The maximization problem is:

$$\max_{Q_i} \pi_i = U(Q_i) + \beta x^* - t - cQ_i. \quad (19)$$

From the first order condition it follows directly the expressions for investment decisions:

$$c = \frac{\gamma^2 - 1}{(1 - r\gamma)^2} Q + \frac{-\gamma}{(1 - r\gamma)} \beta + \frac{1 - \gamma + r - r\gamma}{(1 - r\gamma)^2}, \quad (20)$$

$$c = \frac{\gamma^2 - 1}{-2r\gamma + r^2 + 1} Q + \frac{r - \gamma}{-2r\gamma + r^2 + 1} \beta + \frac{1 - \gamma + r - r\gamma}{-2r\gamma + r^2 + 1}, \quad (21)$$

for the zero rating and joint billing case, respectively. It is evident that the negative effect of increasing the strength of the network effect still persists in the zero rating case, and for joint billing when  $\gamma > r$ . The effect that should be highlighted is that increased capacity has a negative impact on the consumption of  $x$  when  $\gamma > 0$ , which leads to lower marginal benefit of increasing capacity and therefore, less incentives to invest in capacity. Conversely, when  $\gamma < 0$ , the incentives to invest increase with the strength of the network, meaning that Proposition 2 still holds in the monopoly case. However, the effect of increasing the strength of the network effect is always more positive for the joint billing case, as  $\frac{-\gamma}{(1-r\gamma)} < \frac{r-\gamma}{-2r\gamma+r^2+1}$  for any  $\gamma \in ]-1, 1[$ .

Again, it is impossible to compare curves (20) and (21) analytically, and therefore, I will set values for the parameters similar to the competitive case.

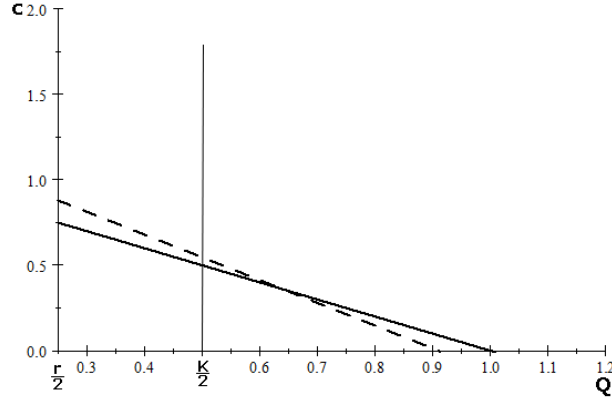


Figure 2: Investment curves with parameters:  $\gamma = 0.5$ ,  $r = 0.5$ ,  $\beta = 0.18$  and  $t = 2$ ; Dashed line: Zero-rating; Solid line: Joint billing

For the given parameter values, on the eligible region for the analysis, capacity investment will always be higher under zero rating (represented by the dashed line). However, looking at figure 6 in appendix B, the opposite is true. By analysing figures 2, 6, 7 and 8, it is possible to see the effects of network effect strength on incentives to invest described above.

Even though the monopoly case yields similar results, it is important to keep in mind that increasing capacity always leads to higher fixed fees and therefore, net utility will remain constant. This means that surplus from consumption increases with more capacity, but overall net utility remains unchanged, leading to consumers being indifferent between levels of capacity. This is not true in the competitive case, as only a fraction of the increase in surplus is captured by the firm.<sup>11</sup> The remainder is kept by the consumers and translates into higher net utility and hence, higher market share for the firm.

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<sup>11</sup>This is evident in equation (11).



## VI Conclusion

Both the competitive and monopoly versions of the model analysed point to the same conclusion: MNOs' argument that zero rating increases investment incentives is not straightforward. In fact, with low enough investment costs, joint billing will lead to higher capacity. While this goes against previous literature where low costs were necessary to increase capacity under zero rating, the overall conclusion that there is no clear increase in capacity when MNOs choose to zero rate holds. Nonetheless, it is important to keep in mind that while neither network effects nor price discrimination (Inceoglu and Liu (2017)) is enough to sustain MNOs' argument unconditionally, there is no study that combines the two (or more) effects identified. Moreover it is not clear in which direction should incentives go, that is, if for low investment costs, the increased incentives from price discrimination outweigh the disincentives from network effects or vice-versa, and the same for high investment costs.

In terms of regulatory needs, this paper and previous literature seem to point to the fact that there is no clear argument pro or against zero rating in terms of total welfare. As such, regulatory agencies should abstain from ex-ante regulation and analyse zero rating in a case by case framework.

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# Appendix A

## Firm's problems

$$\begin{aligned}
\max_{k_i, w_i} \pi_i &= \alpha (S_i(k_i) - w_i) - cQ_i \\
s.t. \quad Q_i &= \alpha k_i \\
(3).
\end{aligned}
\tag{22}$$

FOCs:

$$\begin{aligned}
\frac{\partial \pi}{\partial k_i} &= 0 \Leftrightarrow \frac{\partial S}{\partial k_i} = \mu_i, \\
\frac{\partial \pi}{\partial w_i} &= 0 \Leftrightarrow \frac{1}{2t}(S(k_i) - w_i) - \alpha = \frac{1}{2t}\mu_i k_i.
\end{aligned}$$

Solving for  $w_i$  gives the reaction function for  $w_i$ :

$$w_i = -\frac{t}{2} + \frac{1}{2}S(k_i) + \frac{1}{2}w_j - \frac{1}{2}\frac{\partial S}{\partial k_i}k_i.$$

The same for firm  $j$ :  $w_j = -\frac{t}{2} + \frac{1}{2}S(k_j) + \frac{1}{2}w_i - \frac{1}{2}\frac{\partial u}{\partial k_j}k_j$ . Substituting  $w_j$  in the previous equation gives:

$$\begin{aligned}
w_i &= -\frac{t}{2} + \frac{1}{2}S(k_i) + \frac{1}{2}\left(-\frac{t}{2} + \frac{1}{2}S(k_j) + \frac{1}{2}w_i - \frac{1}{2}\frac{\partial u}{\partial k_j}k_j\right) - \frac{1}{2}\frac{\partial u}{\partial k_i}k_i \Leftrightarrow \\
&\Leftrightarrow w_i = -t + \frac{2}{3}S(k_i) + \frac{1}{3}S(k_j) - \frac{2}{3}\frac{\partial u}{\partial k_i}k_i - \frac{1}{3}\frac{\partial u}{\partial k_j}k_j.
\end{aligned}
\tag{23}$$

## Capacity Choice

### Zero-Rating

Letting  $b = \frac{R}{1-r\gamma}$  and using equation (15), firms' profits are:

$$\begin{aligned}
\max_{k_i} \alpha & \left( \alpha (2t - b) + 2k_1AK - 2Ak_1^2 \right) - c\alpha k_1 \\
s.t. \quad \alpha &= \frac{3t + Ak_1^2 - Ak_2^2 - 2b + b\gamma k_2}{6t - 4b + b\gamma k_1 + b\gamma k_2}.
\end{aligned}$$

With symmetry ( $k_i = k_j = k$  and  $\alpha = \frac{1}{2}$ ) FOC becomes:

$$\begin{aligned} \frac{\partial}{\partial k_i} (\alpha(2t\alpha - b - 2Ak_i^2 + 2AKk_i + 2b\gamma k_i) + \alpha b(1 - \gamma k_i) - c\alpha k_i) = \\ = \frac{1}{4} \frac{\gamma b^2 - 6\gamma b k^2 A - \gamma b k c + 2\gamma b k A K - 2\gamma b t + 12b A k - 8b A K + 4b c - 8A^2 k^3 + 8k^2 A^2 K - 4k^2 c A - 16t A k + 12A K t - 6c t}{3t - 2b + b\gamma k} = 0. \end{aligned}$$

Solving for  $c$ :

$$c = \frac{-8A^2 k^3 + (8A^2 K - 6\gamma b A) k^2 + (2\gamma b A K + 12b A - 16A t) k + (12A K t - 8b A K + \gamma b^2 - 2\gamma b t)}{6t + 4k^2 A + b\gamma k - 4b}.$$

### Joint Billing

Letting  $v = \frac{\beta}{r^2 - 2r\gamma + 1}$  and using equation (16), profit function is then:

$$\begin{aligned} \max_{k_i} \alpha (\alpha (2t - v + rv) + 2k_1 G K - 2G k_1^2) - c\alpha k_1 \\ s.t. \alpha = \frac{3t - 2v + G k_1^2 - G k_2^2 + 2rv - r v k_2 + v\gamma k_2}{6t - 4v + 4rv - r v k_1 - r v k_2 + v\gamma k_1 + v\gamma k_2}. \end{aligned}$$

With symmetry ( $k_i = k_j = k$  and  $\alpha = \frac{1}{2}$ ) FOC:

$$\begin{aligned} \frac{d}{dk_1} (\alpha (\alpha (2t - v + rv) + 2k_1 G K - 2G k_1^2) - c\alpha k_1) = \\ = -\frac{1}{4} \frac{1}{-3t + 2v - 2rv - kv\gamma + krv} (-rv^2 + v^2\gamma - 8G^2 k^3 + r^2 v^2 - 6ct + 4cv - 2tv\gamma \\ - 4Gck^2 - rv^2\gamma + 8G^2 K k^2 + 12G K t - 8G K v - 16G k t + 12G k v - 4c r v + 2r t v - c k v \gamma \\ + 6G k^2 r v - 6G k^2 v \gamma + 8G K r v - 12G k r v + c k r v + 2G K k v \gamma - 2G K k r v) = 0. \end{aligned}$$

Solving for  $c$ :

$$c = \frac{(\gamma - r + r^2 - r\gamma)v^2 - 8G^2 k^3 + (12Gk + 2rt + 6Gk^2 r + 8GKr + 2GKk\gamma - 12Gkr - 2t\gamma - 8GK - 6Gk^2\gamma - 2GKkr)v + 8G^2 K k^2 + (12GK - 16Gk)t}{6t - 4v + 4Gk^2 + 4rv + kv\gamma - krv}.$$

## Appendix B

Dashed lines represent investment under zero rating and solid lines represent investment under joint billing. Only eligible part of the curves are shown, i.e.  $k > r$  or  $Q > \frac{r}{2}$ .

### Competitive case

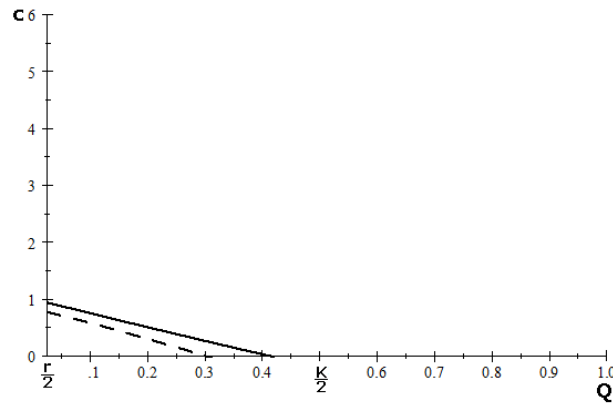


Figure 3: Investment curves with parameters:  $\gamma = 0.5$ ,  $r = 0.5$ ,  $\beta = 1.5$  and  $t = 2$ ; Dashed line: Zero-rating; Solid line: Joint billing

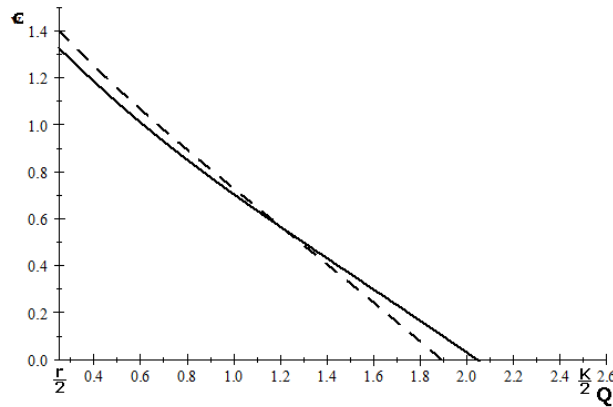


Figure 4: Investment curves with parameters:  $\gamma = -0.5$ ,  $r = 0.5$ ,  $\beta = 1.5$  and  $t = 2$ ; Dashed line: Zero-rating; Solid line: Joint billing

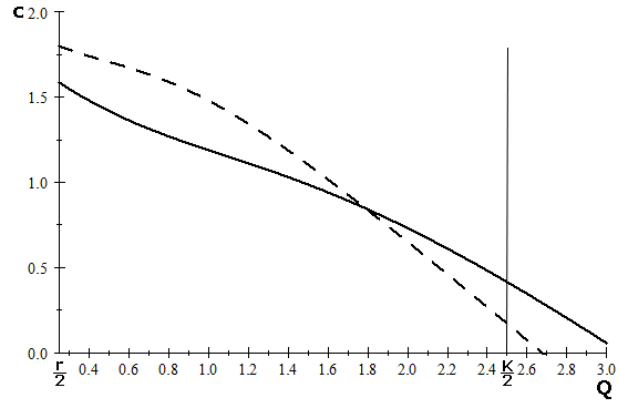


Figure 5: Investment curves with parameters:  $\gamma = -0.5$ ,  $r = 0.5$ ,  $\beta = 2.5$  and  $t = 2$ ; Dashed line: Zero-rating; Solid line: Joint billing

## Monopoly case

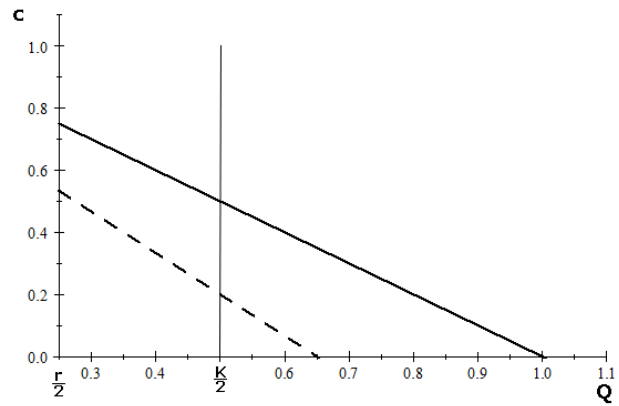


Figure 6: Investment curves with parameters:  $\gamma = 0.5$ ,  $r = 0.5$ ,  $\beta = 0.7$  and  $t = 2$ ; Dashed line: Zero-rating; Solid line: Joint billing

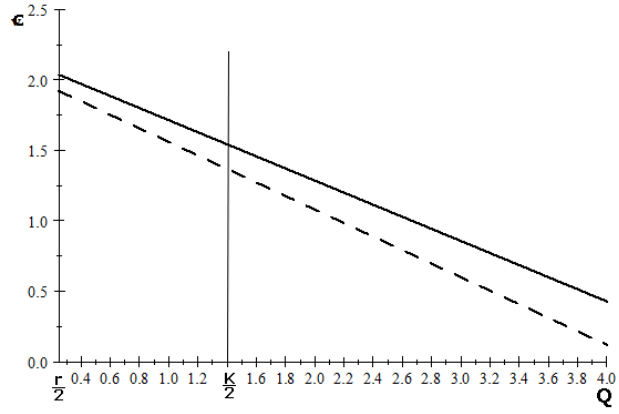


Figure 7: Investment curves with parameters:  $\gamma = -0.5$ ,  $r = 0.5$ ,  $\beta = 1.5$  and  $t = 2$ ; Dashed line: Zero-rating; Solid line: Joint billing

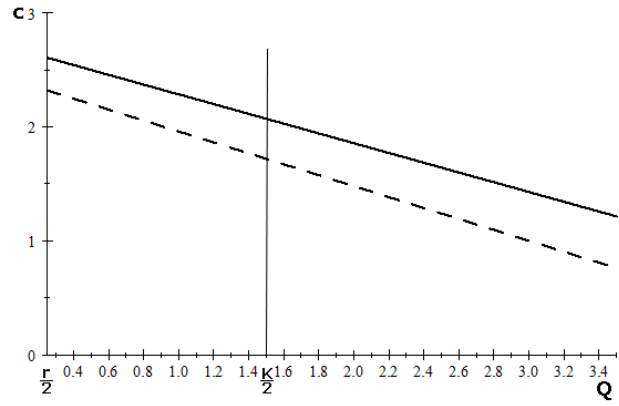


Figure 8: Investment curves with parameters:  $\gamma = -0.5$ ,  $r = 0.5$ ,  $\beta = 2.5$  and  $t = 2$ ; Dashed line: Zero-rating; Solid line: Joint billing